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tion of the galvanometer ensues. The heating current is divided between the two strips, and by suitable resistance coils the circuit is adjusted once for all so that whatever the strength of the heating current it produces equal dissipation of energy in the two strips. If now after closing the shutter the heating current is graduated until the deflection formerly produced by radiation is reproduced by electrical heating, the energy dissipated in either strip is the measure of the absorbed radiation. In the two strip pyranometer the secondary deflection by indirect heating is unimportant, because of the symmetry of the arrangement. However, to avoid this source of error altogether the exposure is limited to 30 seconds, and a full minute is allowed to lapse before introducing electric heating.

Numerous measurements of the sky-radiation have been made from the North Tower of the Smithsonian Institution. On fine days the sky-radiation alone received on a horizontal surface ranges from 0.07 to 0.13 calories per square centimeter per minute. On cloudy days, not thick enough for rain, the values run from 0.20 to 0.30 calories according to the kind of cloudiness prevailing. Measurements were made on the reflection from new fallen snow, and for total solar and sky radiation this proved to be 70%.

In the simpler form the instrument is so sensitive that it could be used in the deep shade of a forest, or with screens of selective transmission, so that it would be suited to botanical as well as meteorological investigations. As in the case of the silver disk pyrliometer, the Smithsonian Institution may undertake to prepare pyranometers at cost (approximately \$150) where valuable investigations may be promoted thereby.

NOTE ON LUCAS' THEOREM

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In a recent note¹ I ventured to give a proof, which I thought might be new, of Lucas' Theorem and one of its more immediate generalizations to rational functions. Professor Bôcher has kindly called my attention to the fact that the same proof had previously been given by him² and that the extension to rational functions was a special case (in the method of proof as well as in the results obtained) of other of his results.³ I called attention to the interesting fact that this proof of Bôcher's applies without modification at once to integral functions of class zero. It is the purpose of this second note to show that it also applies without modi-

fication to *any* integral function of finite class p whose Weierstrass primary factors are of the normal form

$$f(z) = \Pi_{\nu} \left(1 - \frac{z}{\alpha_{\nu}} \right) e^{G_{\nu}(z)},$$

where

$$G_{\nu} = \sum_1^p \frac{1}{n} \left(\frac{z}{\alpha_{\nu}} \right)^n,$$

and where the α 's are all real, or more generally to the integral functions

$$\phi(z) = f(z) \exp(\gamma_1 z^{p+1} + \gamma_2 z^p),$$

where γ_1 and γ_2 are real and γ_1 is negative if p is odd and γ_2 positive or zero if p is even. The essential thing in Bôcher's proof is that the

$$\frac{\phi'}{\phi} = z^p \left[\sum \frac{\alpha_{\nu}^{-p}}{z - \alpha_{\nu}} + \gamma_1 + \gamma_2 z^{-1} \right]$$

vectors which occur in the square brackets all lie inside an angle of 180° and hence their sum cannot vanish if z is not on the axis of reals.

Thus we obtain a much more general result than that obtained by Borel⁴ or Polya⁵ who employed methods that seem applicable only to functions of class zero and one, the theorem at which we have arrived being this: *All the zeros of the derivative of the integral function*

$$e^{\gamma_1 z^{p+1} + \gamma_2 z^p} \Pi \left(1 - \frac{z}{\alpha_{\nu}} \right) e^{G_{\nu}(z)}$$

*are real, if the α 's and γ 's are real and γ_1 is negative, if p is odd and γ_2 positive if p is even.*⁶

¹ These PROCEEDINGS, 2, 247 (1916).

² Bôcher, Some Propositions Concerning the Geometric Representation of Imaginaries, *Annals of Mathematics*, 1892.

³ Bôcher, A Problem in Statics and its Relation to certain Algebraic invariants, *Proc. Amer. Acad. Arts Sci.*, 40, No. 11, 1904. Neither of these papers by Bôcher was listed in the bibliographies of Féjer and Hyashi, which I cited, and hence were overlooked by me.

⁴ Fonctions Entières, pp. 32 et seq.

⁵ Bemerkung zur Theorie der Ganzen Funktionen, *Jahresber. D. Math.-Ver.*, October-December, 1916.

⁶ The same proof can be further applied to

$$\psi(z) = f(z) \exp. (\gamma_0 z^{p+2} + \gamma_1 z^{p+1} + \gamma_2 z^p).$$